Eco514—Game Theory
Lecture 16: Applications of Sequential and Perfect Bayesian Equilibrium

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Introduction

The purpose of this lecture is to help you familiarize with the workings of sequential equilibrium and "sequential equilibrium lite", i.e. perfect Bayesian equilibrium.

The main focus is the "reputation" result of Kreps and Wilson (1982). You should refer to OR for details and definitions: I am following the textbook quite closely.

We have already mentioned the Entry Deterrence game. Now consider a K-fold repetition of the game in which an incumbent faces a sequence of K (potential) entrants. It is clear that, in the subgame in which the last entrant gets a chance to play, the incumbent will Concede. But then, by backward induction, he will also Concede at any previous round.

This conclusion seems unpalatable. One would expect the chain-store to fight a few times early on so as to deter further entry; yet, this simply cannot occur in equilibrium.

The Chain-Store Game with Imperfect Information

Kreps and Wilson suggest the following way out of the Chain Store Paradox. Assume that, with some small probability $\epsilon > 0$, the incumbent is "Tough": that is, she really enjoys beating up entrants.

Consider the behavior of a "Regular" incumbent; clearly, when she faces the last entrant, she will Concede if the latter moves In. However, in previous rounds, she may threaten to Fight entry; in this case, if the *current* entrant moves In and is met with a fight, the *following* entrant does not learn whether the incumbent is Regular or Tough, because both Fight entry.

Hence, the posterior probability that the incumbent is Tough remains constant at ϵ, and if this value is such that the following Entrant prefers not to Enter, the incumbent's threat is credible.

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Things are not quite so simple in actuality, but Kreps and Wilson prove that something very close to this can be supported in a sequential equilibrium; indeed, while there exist other sequential equilibria in which no "reputation" is maintained, there exists a simple restriction on the entrants' beliefs which ensures that the reputation outcome obtains.

OR analyze the situation using the notion of PBE, and do not investigate outcome uniqueness. I will follow their treatment, emphasizing certain key steps.

Model

Payoffs in the stage game are as follows:

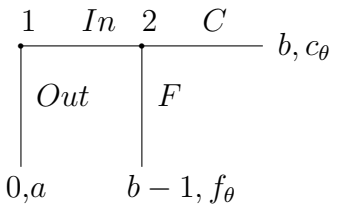

Figure 1: $\Theta = \Theta_1 = \{R, T\}$, $a > 1 > b > 0$, $c_R = f_T = 0$, $f_R = c_T = -1$.

To understand the equilibrium strategies, observe first that, if the entrant expect the incumbent to Fight with probability b, her expected payoff from entry equals $b(1-b) + (b-1)b = 0$; hence, she is indifferent between staying Out and going In.

Now, the Tough incumbent will always Fight in the PBE under consideration. This implies, among other things, that as soon as an entrant observes C in any previous round, he will conclude that the incumbent is Regular, and therefore will enter; so will every subsequent entrant.

Consider the problem faced by the Regular incumbent who observes entry at some stage $k < K$. If she Concedes, she gets 0 in the current period, but faces entry thereafter—and hence, by best-responding, she can secure a payoff of no more than 0. Now suppose that she Fights, and that this leads the next entrant to revise her beliefs in such a way that he is indifferent between In and Out; suppose further that, this being the case, the next entrant chooses Out with probability $\frac{1}{a}$. Then, by Fighting the current entrant (and best-responding thereafter), the incumbent also gets a continuation payoff of 0.

Thus, following entry, the incumbent is indifferent between F and C. Hence, he can randomize, and the probability of C can be chosen so that the *posterior* beliefs of the next entrant will be such that he is indeed indifferent between going In and staying Out.

[Notice that this argument relies heavily on the equilibrium assumption!]

Equilibrium

[Refer to OR for definitions]

The basic idea is as follows.

- The first $k^* - 1$ entrants will stay Out, where $k^* = \min\{k : b^K - k + 1 \geq \epsilon\}$. Hence, nothing will be learned about the incumbent's type in equilibrium during the first $k^* - 1$ rounds: $t(h) < k^*$ implies $\mu(h)(T) = \epsilon$.
- If, however, one of the first $k^* - 1$ entrants does enter, the Regular incumbent Fights. Note that this implies that, even in this case, nothing will be learned about the incumbent's type; hence, the continuation play following such a deviation resembles equilibrium play.
- Entrant k^* goes In if $\epsilon = \mu(h)(T) < b^{K-k^*+1}$, and randomizes between In and Out in case of equality. In either case, if there is entry, the Incumbent reacts by randomizing between C and F. If there is no entry, beliefs are unchanged, so the next entrant certainly enters.
- Beliefs are thus updated via Bayes' rule. Thus, as soon as the Incumbent plays C, the following Entrants all choose In. Also, whenever an entrant goes In and the incumbent Fights, the next entrant is indifferent between In and Out, and hence randomizes.
- In the last round, the Incumbent Concedes.

You should convince yourself that the equilibrium described in OR achieves precisely this. In particular, you should see that, whenever $\mu(h)(T) = b^{K-k+1}$, the probability of a Fight in the following round is exactly b (so the next entrant is indeed willing to randomize). Also, the posterior probability of T given F is b^{K-k}, as per the equilibrium beliefs. Finally, as claimed in the previous section, it is easy to see that, following entry at or after Stage k^*, the continuation payoff to the incumbent if he plays F is indeed 0, as it must be to justify randomization.

Plausible Beliefs

The preceding arguments go a long way (but not all the way) towards proving that reputation can be maintained in a PBE. However, the original result due to Kreps and Wilson is stronger: first, they use the notion of Sequential Equilibrium; second, they show that *any SE which satisfies a certain restriction on beliefs induces the reputation outcome.*

The restriction on beliefs has the following form: consider two histories h and h' such that h' differs from h only in that, at some subhistories following entry, the incumbent

Concedes instead of Fighting. Then it must be the case that $\mu(h)(R) \geq \mu(h')(R)$. This makes sense—and Kreps and Wilson deem beliefs which satisfy this condition *plausible*.

The key point is that one could construct "strange" equilibria in which reacting to off-equilibrium entry with F induces subsequent entrants to believe that the incumbent is Regular: this could engender further entry out of equilibrium. If this is the case, the incumbent might Concede early in the game, in order to avoid (!) being identified as a regular type (!!!).

You can convince yourself that the beliefs used in the construction of our "reputational equilibrium" are indeed plausible.

Building vs. Maintaining a Reputation

Note that, in the equilibrium we have constructed, essentially nothing happens in the first $k^* - 1$ stages. Thus, there is no sense in which the incumbent "builds a reputation" for being tough. This can only be said of the last few stages—but note that the length of the randomization phase only depends on ϵ, and not on the overall length of the game. Thus, one may have a very protracted play phase in which no reputation is built, and a comparatively tiny one in which actual reputation building occurs.

Rather, this story has to do with *maintaining* a reputation for toughness. Also, it should be apparent that the result depends crucially on the equilibrium assumption: consider the delicate interlocking of randomizations.